Integers

2

A universal language?

Mathematics might be used to communicate with aliens. How would this work?

How could we use maths to discover other forms of intelligent life in the universe? Our number system is based on tens (mainly because we have ten fingers), but we cannot assume that an alien number system would be the same.

It is believed that the best way to send a universal message might be to use prime numbers. Prime numbers, such as 2, 3, 5 and 7, have only two factors: 1 and the number itself. This property means that prime numbers will be the same in any number system.

In 1974, the Arecibo telescope in Puerto Rico broadcast a message into a star cluster 21 000 light years away. The message consisted of 1679 'bits' of data, which can be arranged into 73 lines of 23 characters (73 and 23 are prime numbers). No answer has been detected yet; this is not surprising given the distance it will have to travel. Later in this chapter you can learn about another way prime numbers are used to send information.

Forum

If you had the opportunity to send the first message to an alien species, what would you say?

Our number system is based on multiplying and dividing by 10. However, sometimes we count by 2, 12, 60, 360 and 365. What do we count using these numbers?

Why learn this?

Understanding relationships between numbers allows you to work with them confidently and efficiently, often without the need for a calculator. A knowledge of factors, multiples and prime numbers is a good foundation for study of many other areas of mathematics. Negative numbers are another important set of numbers,used for example when working with temperatures, elevations, score differences and money.

After completing this chapter you will be able to:

- find the lowest common multiple of a group of numbers
- find the highest common factor of a group of numbers
- use divisibility tests to assist in finding factors
- identify prime and composite numbers
- find the prime factors of a number
- use positive and negative numbers (integers) to represent quantities
- compare and order integers
- add and subtract integers.

Recall 2

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 Copy and complete these within 3 minutes.

(a)	$6 \times 7 =$	$6 \times 6 =$	$6 \times 4 =$	$6 \times 11 =$	$6 \times 8 =$
(b)	$7 \times 11 =$	$7 \times 7 =$	$7 \times 5 =$	$7 \times 2 =$	$7 \times 3 =$
(c)	$8 \times 7 =$	$8 \times 6 =$	$8 \times 4 =$	$8 \times 10 =$	$8 \times 8 =$
(d)	$9 \times 12 =$	$9 \times 3 =$	$9 \times 5 =$	$9 \times 11 =$	$9 \times 8 =$
(e)	$12 \times 7 =$	$12 \times 6 =$	$12 \times 12 =$	$12 \times 9 =$	$12 \times 11 =$

2 **(a)** List all the digits that an even number can end with.

(b) List all the digits that an odd number can end with.

3 Copy and complete each of the following by writing a < (less than) or > (greater than) symbol between the given values.

(a) 10 _____ 7 **(b)** 3 _____ 6 **(c)** 2 _____ 0 **(d)** 0 _____ 5

4 Calculate:

(a) $3 + 8 + 12$ **(b)** $22 + 19 - 7$ **(c)** $22 - 9 + 87 - 35$

(d) $18 - 9 - 4$ **(e)** $72 - 39 + 14$ **(f)** $51 + 43 - 11 - 7$

5 Write the following temperatures in order from coldest to warmest.

(a) 15 °C, 7 °C, 0 °C, -4 °C, 21 °C, -11 °C

(b) 5 °C, -3 °C, 10 °C, -25 °C, 32 °C, -14 °C

6 Write the following in expanded form, then evaluate.

(a) 7^2 **(b)** 3^4 **(c)** 2^6 **(d)** 1^9

7 Calculate the following.

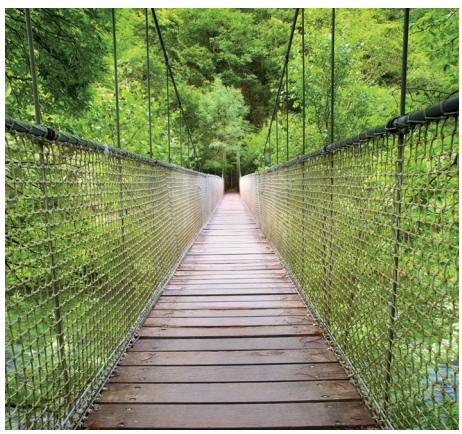
(a) $3^2 \times 5^2$ **(b)** $4^3 \div 2^3$ **(c)** $8^2 + 6^2$ **(d)** $9^2 - 7^2$

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Is adding always more? Is subtracting always less?

In this activity, you will explore the relationship between the addition and subtraction of negative numbers and the direction of the number line.



Multiples, factors and divisibility

2.1

Multiples and factors

The numbers 1, 2, 3, 4, 5, … are often called whole numbers or counting numbers. (When … is written in mathematics, this means that the pattern goes on forever.)

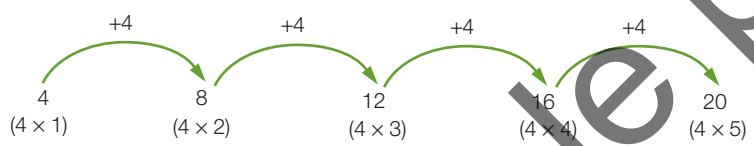
You can find the **multiples** of a whole number by multiplying it by another whole number.

For example, the multiples of 7 are:

	1×7	2×7	3×7	4×7	5×7	…
Multiples of 7	7	14	21	28	35	…

Another way to create a list of multiples of a number is to start with the number and add it repeatedly.

For example, the multiples of 4 are:



The first in the sequence of multiples of a number is always the number itself. You can see from the above table and sequence that the first multiple of 7 is 7 (1×7), and the first multiple of 4 is 4 (1×4).

A **factor** is a number that divides exactly into another number.

'Exactly' means that there is no remainder left after the division.

You can think of the process of finding factors as the reverse of finding multiples.

By reversing (flipping) the above table, you can see some factors:

	7	14	21	28	35	…
Some factors	1, 7	2, 7	3, 7	4, 7	5,7	…

This means that the factors of 7 are 1 and 7, some factors of 14 are 2 and 7 etc.

It is often important to find *all* the factors that a number has. You can see from the table that 28 has factors of 4 and 7, because 4 and 7 multiply to give 28.

However, 28 has other factors as well:

$28 = 4 \times 7$
and $28 = 2 \times 14$
and $28 = 1 \times 28$

So, 28 has six factors: 1, 2, 4, 7, 14 and 28.

Worked example 1 W.E. 1

Find all the factors of each of the following numbers.

(a) 12 **(b)** 110

Thinking		Working
(a) **1**	Write the pairs of numbers that multiply to give the original number. The number will always be divisible by 1, so write 1 × original number as the first pair, then consider whether there are pairs beginning with 2, 3 etc.	(a) $1 \times 12 = 12$ $2 \times 6 = 12$ $3 \times 4 = 12$
2	List the factors from smallest to largest.	Factors of 12: 1, 2, 3, 4, 6, 12.
(b) **1**	Write the pairs of numbers that multiply to give the original number. The number will always be divisible by 1, so write 1 × original number as the first pair, then consider whether there are pairs beginning with 2, 3 etc.	(b) $1 \times 110 = 110$ $2 \times 55 = 110$ $5 \times 22 = 110$ $10 \times 11 = 110$
2	List the factors from smallest to largest.	Factors of 110: 1, 2, 5, 10, 11, 22, 55, 110.

Sometimes, two of the same factor are multiplied to give the original number. For example, $7 \times 7 = 49$. You include 7 only once in the list of factors for 49.

Divisibility

Another way of considering factors and multiples is to talk about divisibility. A larger number is **divisible** by a smaller number if dividing by the smaller number gives an exact whole number answer with no remainder. The following sentences all describe the same idea.

- Two factors of 35 are 5 and 7.
- 35 is divisible by 5 and 7.
- Both 5 and 7 go into 35 exactly (without any remainder).
- 5 multiplied by 7 gives 35.
- 35 is a multiple of 5 and also a multiple of 7.

A good knowledge of factors and multiples will help you determine which numbers are divisible by others. For larger numbers, you can use tests to determine whether one number is divisible by another. These tests are summarised in the following table.

A number is divisible by …	… if it passes this divisibility test
2	The number is an even number (ends in 0, 2, 4, 6 or 8).
3	The sum of the digits is divisible by 3.
4	The number formed by the last two digits is divisible by 4.
5	The last digit is 0 or 5.
6	The number is even (divisible by 2) and also divisible by 3.
8	The number formed by the last 3 digits is divisible by 8.
9	The sum of the digits is divisible by 9.
10	The last digit is 0.

Worked example 2

WE 2

Using the divisibility tests, determine which of the numbers 75, 98, 110 and 132 are divisible by each of the following.

(a) 3 **(b)** 4 **(c)** 5 **(d)** 6

Thinking		Working
(a) 1	To test if a number is divisible by 3, add the digits in each of the numbers. If the sum of the digits is divisible by 3, the number is divisible by 3.	*(a)* 75: 7 + 5 = 12 ✓ 98: 9 + 8 = 17 ✗ 110: 1 + 1 + 0 = 2 ✗ 132: 1 + 3 + 2 = 6 ✓
2	State the answer for each number.	75 and 132 are divisible by 3. 98 and 110 are not divisible by 3.
(b) 1	To test if a number is divisible by 4, look at the number formed by the last two digits. If that number is divisible by 4, then the whole number is divisible by 4.	*(b)* 75 ✗ 98 ✗ 110 ✗ 132 ✓
2	State the answer for each number.	132 is divisible by 4. 75, 98 and 110 are not divisible by 4.
(c) 1	To test if a number is divisible by 5, is the last digit 5 or 0?	*(c)* 75 ✓ 98 ✗ 110 ✓ 132 ✗
2	State the answer for each number.	75 and 110 are divisible by 5. 98 and 132 are not divisible by 5.
(d) 1	To test if a number is divisible by 6, perform the tests for divisibility of 2 and 3. Write the even numbers (numbers divisible by 2). Add the digits in each of these numbers and see whether the number is divisible by 3.	*(d)* Using the working from *(a)*: 98: 17 ✗ 110: 2 ✗ 132: 6 ✓
2	State the answer for each number.	132 is divisible by 6. 75, 98 and 110 are not divisible by 6.

Multiples of a whole number are found by multiplying it by another whole number.

A factor is a number that divides exactly into another number.

Divisibility tests can help find the factors of a whole number.

Common multiples

A **common multiple** of two numbers is a number that both of them divide into exactly. (In other words, the multiple is 'common' to both of them.) Changing the multiple table from the start of the section slightly, you get:

	1 and 7	2 and 7	3 and 7	4 and 7	5 and 7	...
A common multiple	7	14	21	28	35	...

This table only gives one common multiple for each pair of numbers. There are an infinite number of others. The **lowest common multiple (LCM)** of two numbers is the *smallest* number that both of the numbers divide into exactly. The common multiples of 2 and 7 are 14, 28, 42, 56,… so the LCM of 2 and 7 is 14. Because numbers can always be higher, there is no such thing as a highest common multiple.

Worked example 3 W.E. 3

Find the lowest common multiple (LCM) of the following numbers, by first listing the multiples of each: 4 and 6.

Thinking	Working
1 List the first few multiples of the first number.	4: 4, 8, (12), 16, 20, 24, …
2 List the first few multiples of the second number.	6: 6, (12), 18, 24, 30, 36, …
3 Circle the first number that appears in both lists. This is the LCM.	LCM of 4 and 6 is 12.

Common factors

A **common factor** of two numbers is a number that divides exactly into both of them. Common factors should not be confused with common multiples. Consider the following.

	7 and 14	4 and 20	9 and 15	8 and 40	12 and 18
Common factors	1, 7	1, 2, 4	1, 3	1, 2, 4, 8	1, 2, 3, 6

The number 1 will always be a common factor of any set of numbers.

It is often useful to find the **highest common factor (HCF)** of a pair of numbers. From the above table, you can see that the HCF of 7 and 14 is 7, the HCF of 9 and 15 is 3, the HCF of 12 and 18 is 6 etc.

If the smaller number in the pair is a factor of the larger number, then the smaller number is the HCF. For example, the HCF of 4 and 20 is 4. The HCF of 8 and 40 is 8. The HCF of a pair of numbers cannot be bigger than the smaller number of the pair.

Worked example 4 W.E. 4

Find the highest common factor (HCF) of the following pairs of numbers, by first listing the factors of each number: 12 and 18.

Thinking	Working
1 List all factors of the first number.	12: 1, 2, 3, 4, 6, 12
List all factors of the second number.	18: 1, 2, 3, 6, 9, 18
2 Circle the factors appearing in both lists. These are the common factors.	
3 Select the largest number that appears in both lists. This is the HCF.	HCF of 12 and 18 is 6.

The lowest common multiple (LCM) of two numbers is the smallest number that both of the numbers divide into exactly.

The highest common factor (HCF) of two numbers is the largest number that divides exactly into both of the numbers. The highest common factor is sometimes also called the greatest common divisor (GCD).

2.1 Multiples, factors and divisibility

Navigator

1 (columns 1–2), 2, 3, 4 (columns 1–2), 5 (a–h), 6, 7, 8, 9, 10, 11, 12, 13, 14 (a), 15, 16, 17, 19 (a–b), 21, 22, 25, 26, 28	1 (columns 2–3), 2, 3, 4 (columns 2–3), 5 (columns 2–3), 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19 (a–c), 21, 22, 23, 25, 26, 28	1 (columns 3–4), 3, 4 (j–o), 5 (i–l), 7, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28

Answers p. 652

Fluency

1 Find all the factors of each of the following numbers. W.E. 1

(a) 18 (b) 16 (c) 23 (d) 24

(e) 20 (f) 35 (g) 36 (h) 42

(i) 53 (j) 60 (k) 77 (l) 84

2 How can you check whether a number is divisible by the following?

(a) divisible by 2 (b) divisible by 10 (c) divisible by 3

(d) divisible by 6 (e) divisible by 4 (f) divisible by 5

3 Determine which of the numbers 92, 108, 245 and 3100 are divisible by each of the following. W.E. 2

(a) 3 (b) 4 (c) 5 (d) 8 (e) 9

W.E. 3

4 Find the lowest common multiple (LCM) of the following sets of numbers, by first listing the multiples of each.

(a) 2 and 5 **(b)** 3 and 9 **(c)** 5 and 25
(d) 5 and 6 **(e)** 4 and 7 **(f)** 8 and 12
(g) 7 and 9 **(h)** 10 and 12 **(i)** 6 and 11
(j) 9 and 12 **(k)** 20 and 50 **(l)** 8 and 14
(m) 3, 4 and 5 **(n)** 2, 25 and 50 **(o)** 20, 50 and 60

W.E. 4

5 Find the highest common factor (HCF) of the following pairs of numbers, by first listing the factors of each number.

(a) 10 and 15 **(b)** 8 and 24 **(c)** 5 and 12 **(d)** 26 and 36
(e) 11 and 33 **(f)** 28 and 70 **(g)** 44 and 22 **(h)** 10 and 30
(i) 40 and 70 **(j)** 32 and 60 **(k)** 35 and 70 **(l)** 42 and 48

6 **(a)** The lowest common multiple of 8 and 1 is:

A 8 **B** 16 **C** 24 **D** 80

(b) Which of the following is a factor of 34?

A 4 **B** 12 **C** 17 **D** 68

7 **(a)** A number divisible by 2, 3 and 5 is:

A 6 **B** 15 **C** 60 **D** 65

(b) Which pair of numbers are both divisible by 4?

A 38 and 42 **B** 38 and 52 **C** 38 and 60 **D** 52 and 60

8 Which list of numbers only has multiples of 30?

A 31, 32, 33, 34, 35 **B** 30, 40, 50, 60
C 30, 60, 90, 120 **D** 1, 2, 3, 5, 6, 10, 15, 30

Understanding

9 **(a)** Which one of the following numbers is not a multiple of 8?

A 4 **B** 24 **C** 72 **D** 88

(b) Which of the following is not a factor of 42?

A 1 **B** 6 **C** 21 **D** 84

10 How many factors does the number 18 have?

A 2 **B** 3 **C** 5 **D** 6

11 State true (T) or false (F) for the following.

(a) 346 is a multiple of 3. **(b)** 872 is divisible by 6.
(c) 2 is a factor of 348. **(d)** 52 is a multiple of 4.
(e) 854 is divisible by 9. **(f)** 3 is a factor of 56 902.

12 For each group of numbers, find **(i)** the LCM and **(ii)** the HCF.

(a) 4, 6 and 10 **(b)** 6, 8 and 12
(c) 8, 12 and 16 **(d)** 10, 25 and 40

13 Complete the following sentences by using the words 'multiple', 'factor' or 'divisible'.

(a) 32 is a multiple of 8 because it is _____ by 8.

(b) 6 is a _____ of 54, so 54 is a multiple of 6.

(c) 72 is divisible by 9, so that makes it a _____ of 9.

(d) 4 is a factor of 60, so 60 is _____ by 4.

14 **(a)** If 24 lollies are placed into bags so that each bag contains the same number, how many lollies can be in each bag? List all possible answers.

(b) If 36 lollies are placed into bags so that each bag contains the same number, how many lollies can be in each bag? List all possible answers.

15 Ms Williams wants to arrange the seating in the hall for the Year 7s. There must be the same number of chairs in each row. She wants the students to take up all the seats in a row. There are 96 students.

(a) How many rows could there be, and how many seats are in each row? Give all possible combinations, including impractical ones.

(b) Ms Williams would like the arrangement to be as 'square' as possible. Which arrangement is best for this?

16 Mr Rasheed is putting his students into groups to work on a project. Students must be in groups of 3 or 4. He has 26 students in his class. Find the two different ways Mr Rasheed can divide up his class.

17 The smallest number divisible by 3, 4 and 5 is:

A 12 **B** 24 **C** 30 **D** 60

18 In a lighting display, one light flashes every 25 seconds while another light flashes every 60 seconds. If both lights are turned on at the same time, then write the next three times when both lights will flash together. (Hint: The lights will flash together at a common multiple of their flashing times.)

19 **(a)** Find the lowest number greater than 50 that is divisible by 7.

(b) Find the lowest number greater than 100 that is divisible by 11.

(c) Find the first common multiple of 2 and 7 that is greater than 100.

(d) Find the first common multiple of 2, 5 and 7 that is greater than 200.

Reasoning

20 Peter power-walked around an oval while Mei Ling jogged. They started and finished at the same time. They started on the same spot and went in the same direction, keeping up a constant speed for 1 hour. Peter walked 8 laps and Mei Ling jogged 24 laps in the hour.

(a) How many times did Mei Ling pass Peter?

(b) How many times did Mei Ling pass Peter exactly on the spot where they started?

(c) At the beginning of which laps did Mei Ling pass Peter exactly on the spot where they started?

21 **(a)** Copy the following table and do the divisibility tests on the numbers in the left column. Circle the number if the original number is divisible by it. The first one has been done for you.

	Divisible by…							
100 000	(2)	3	(4)	(5)	6	(8)	9	(10)
202 008	2	3	4	5	6	8	9	10
12 121 212	2	3	4	5	6	8	9	10
300 300 300	2	3	4	5	6	8	9	10
7 500	2	3	4	5	6	8	9	10
900 090	2	3	4	5	6	8	9	10
123 456 789	2	3	4	5	6	8	9	10

(b) Complete the following.

(i) If a number is divisible by 4, then it is also divisible by _____.

(ii) If a number is divisible by 9, then it is also divisible by _____.

(c) Explain your answers to **(b)**.

22 To test whether a number is divisible by 6, you must test whether it is divisible by 2 *and* 3. Explain why this test works.

23 A 'perfect' number is a number that has the sum of its factors (excluding itself) equal to itself. The first perfect number is 6, because its factors are $1 + 2 + 3 = 6$.

(a) What is the next perfect number? It is less than 40.

(b) The next perfect number is between 490 and 510. See if you can find it.

24 An 'abundant' number is a number for which the sum of its factors is greater than two times the number itself. The first abundant number is 12, as $1 + 2 + 3 + 4 + 6 + 12 = 28$, which is greater than 2×12. Find the next two abundant numbers. (Both are less than 40.)

25 **(a)** How can you always find a common multiple of a pair of numbers?

(b) How can you check if this number is the lowest common multiple?

Open-ended

26 Darren is designing a box for 60 identical chocolates to be placed in rows.

(a) Find three ways Darren could arrange the chocolates in the box.

(b) Which of your arrangements do you think is the most practical for a chocolate box? Explain your answer.

27 Zena is five years of age and Sam is less than 90 years old. Sam's age is a multiple of three and is also a multiple of Zena's age. Find three possible ages Sam could be.

28 Is it possible to find the highest common multiple of two or more numbers? Explain your answer.

Primes and composites

2.2

A number that has more than two factors is called a **composite number**.

A whole number greater than 1 that has exactly two factors, itself and 1, is a **prime number**.

The number 7 is a prime number as its factors are 1 and 7. The number 8 is a composite number as its factors are 1, 2, 4 and 8.

Two numbers are said to be **co-prime** if their highest common factor is 1.

The sieve of Eratosthenes

Eratosthenes was a Greek mathematician who lived more than 2200 years ago. He is believed to be the first person to calculate a value for the circumference of the Earth. He is also famous today for his 'sieve', a method for finding prime numbers.

To use the sieve of Eratosthenes, copy the table and follow the instructions.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Step 1 Cross out the number 1.

Step 2 Go to the next number, which is 2, and circle it. Then, cross out all of the other multiples of 2.

Step 3 Go to the next number that isn't crossed out. This should be 3. Circle it. Then, cross out all of the other multiples of 3.

Step 4 Go to the next number that isn't crossed out, circle it, then cross out all of its multiples.

Step 5 Repeat for the next number that isn't crossed out. Keep repeating this process until there is no 'next number'.

Step 6 Write the factors of each of the circled numbers. What types of numbers are these?

Step 7 Write the factors of any five of the crossed out numbers, except for 1.

Step 8 Which type of number—circled or crossed out—has more factors? Explain why.

2.2 Primes and composites

Navigator

Answers p. 653

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14 (a), 16, 17	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18	1, 3, 4 (b), 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18

Use 'The sieve of Eratosthenes' on the previous page to help you answer Questions **1–6**.

Fluency

1 Write the prime numbers between 1 and 20.

2 How many single-digit prime numbers are there? List them.

3 List all the primes between 20 and 60.

4 **(a)** The first prime number after 50 is:

A 51 **B** 53 **C** 55 **D** 57

(b) A number co-prime with 18 is:

A 9 **B** 21 **C** 24 **D** 25

Understanding

To show that a number is composite, you only need to show that one of the divisibility tests works.

5 **(a)** What is the next prime number after 60?

(b) What is the next composite number after 60?

(c) What are the two odd composite numbers less than 20?

(d) What is the largest prime number less than 50?

6 Write true (T) or false (F) for each of the following statements.

(a) 21 is prime.

(b) 38 is composite.

(c) 59 is prime.

(d) 49 is prime.

(e) 5 and 7 are co-prime.

(f) 5 and 6 are co-prime.

(g) All even numbers greater than 2 are composite.

(h) All prime numbers are odd.

7 Name a divisibility test that shows that the following numbers are composites.

(a) 410 **(b)** 621 **(c)** 9909

(d) 4 516 803 **(e)** 87 912 404 **(f)** 2 871 025

8 Are the following pairs of numbers co-prime? Give reasons for your answer.

(a) 9 and 17 **(b)** 8 and 11 **(c)** 13 and 52 **(d)** 27 and 63

Reasoning

9 Explain why any pair of prime numbers is co-prime.

10 2 is the only even prime number. Explain why.

11 Explain why it is easy to tell that 4 567 278 is a composite number.

12 Explain why 2 and 3 are the only two consecutive prime numbers.

13 What is the smallest difference between any two consecutive composite numbers?

14 **(a)** Find the numbers closest to 100 that are co-prime with 100.

(b) Find the numbers closest to 36 that are co-prime with 36.

15 Will a prime number always be co-prime with any other whole number? Explain your answer.

16 If one number is a multiple of another number and both numbers are greater than 1, explain why they cannot be co-prime.

Open-ended

17 A conjecture is a mathematical statement that is believed to be true, but has not yet been proven. Goldbach's conjecture (named after the mathematician Christian Goldbach) states that 'every even number greater than 2 can be written as the sum of two primes'. Choose 10 even numbers, and use them to demonstrate Goldbach's conjecture.

18 A pair of 'Sophie Germain primes' (named after the mathematician) is a pair of prime numbers where one number is exactly one more than double the other number. For example, 11 and 23 are Sophie Germain primes, because $11 \times 2 + 1 = 23$. Find two more pairs of Sophie Germain primes.

Puzzle

Gold digger

How to play

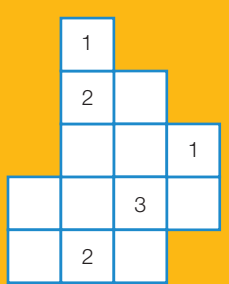
It's the final day of the 16th annual gold-digging competition. Carmen, your partner for the competition, has almost worked out where the gold is located. On two separate maps drawn as square grids, she has marked the squares that have gold nearby.

If a number is written in a grid square on the map, then it has no gold, but there must be gold in that number of the squares touching it (either horizontally, vertically or diagonally, sharing an edge or a corner). So if a square is numbered 3, then there must be gold in 3 of the squares that touch it. No square contains more than one piece of gold.

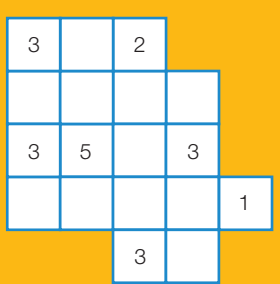
Your task is to find exactly which squares have gold, so your team can get all the gold and win the competition.

Now, copy the following maps and find the gold.

Map (a)



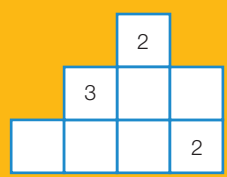
Map (b)



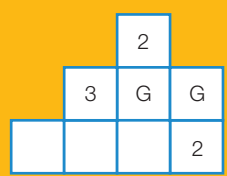
Hints for finding gold

Use the numbered squares to eliminate the possibilities.

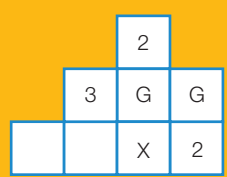
Here, the top square (numbered 2) is only touching two other empty squares, so both of these must contain pieces of gold. Mark these squares with a 'G' to signify this.



Now, look at the 2 in the bottom right-hand corner. It is already next to 2 pieces of gold, so the other square it touches must be empty. Mark this square with an X.



Now, look at the 3. It is already next to one piece of gold, and it is only touching two other empty squares, so both of these squares must contain gold.



2.7 Simplifying addition and subtraction

As seen, you can simplify operations with directed numbers by omitting the '+' sign in front of positive numbers. For example, +7 can be written simply as 7. This can be used to write additions and subtractions more simply.

+4 + (+5) = +9 can be written as 4 + 5 = 9

+4 – (+5) = -1 can be written as 4 – 5 = -1

You can simplify other expressions by replacing the two signs between the numbers with a single symbol.

As seen, adding a negative number and subtracting a positive number are the same. Both result in a movement to the left on the number line; that is, subtraction.

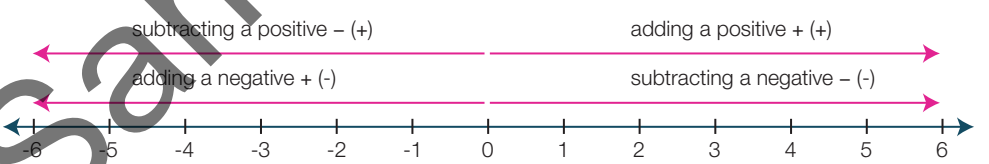
7 + (-3) is the same as 7 – (+3). Both can be written as 7 – 3.

+ (-) and – (+) can be replaced by –

As also seen, adding a positive number and subtracting a negative number are the same. Both result in a movement to the right on the number line; that is, addition.

6 + (+2) is the same as 6 – (-2). Both are equal to 6 + 2.

+ (+) and – (-) can be replaced by +



These observations can be summarised with the following rules for adding and subtracting integers:

If the two signs are the same, the result is addition.
+ (+) = +
– (-) = +

If the two signs are different, the result is subtraction.
– (+) = –
+ (-) = –

To apply these rules, the two signs must be *next* to each other when the calculation is written. The rules do not apply otherwise.

A visual method

Remember you can always use a number line if you do not remember the rules.

1. Draw or imagine a number line. Place your pen, or imagine you are standing, on the first number.
2. Look at the operations symbol:

   If it is +, turn and face right (towards the positive end of the number line).

   If it is –, turn and face left (towards the negative end of the number line).
3. Look at the sign of the second number:

   If it is positive, walk forwards that many units.

   If it is negative, walk backwards that many units.

Worked example 15 W.E. 15

Simplify each of the following by writing a single symbol between the values, then calculate the answer. Use a number line if necessary.

(a) 4 + (+9) **(b)** -1 + (-3) **(c)** 10 – (+1) **(d)** -6 – (-5)

Thinking			Working
(a)	**1**	Adding a positive number is straightforward addition. Replace + (+) with +.	(a) 4 + (+9) = 4 + 9
	2	Calculate.	= 13
(b)	**1**	Adding a negative number is equivalent to subtraction. Replace + (-) with –.	(b) -1 + (-3) = -1 – 3
	2	Calculate.	= -4
(c)	**1**	Subtracting a positive number is straightforward subtraction. Replace – (+) with –.	(c) 10 – (+1) = 10 – 1
	2	Calculate.	= 9
(d)	**1**	Subtracting a negative number is equivalent to addition. Replace – (-) with +.	(d) -6 – (-5) = -6 + 5
	2	Calculate.	= -1

2.7 Simplifying addition and subtraction

Navigator

Answers p. 656

1 (columns 1–2), 2, 3 (a–h), 4, 5 (columns 1–3), 6, 7, 8, 9, 12 (a), 13	1 (columns 2–3), 2, 3 (columns 1–3), 4, 5 (columns 2–3), 6, 7, 8, 9, 10, 12, 13	1 (column 4), 3 (e–l), 5 (column 4), 6, 7, 8, 9, 10, 11, 12, 13

Fluency

W.E. 15

1 Simplify each of the following by writing a single symbol between the values, then calculate the answer. Use a number line if necessary.

(a) +5 + (+2)	**(b)** -1 + (+4)	**(c)** +5 + (+6)	**(d)** -9 + (+7)
(e) +5 – (+3)	**(f)** -8 – (+4)	**(g)** +1 – (+12)	**(h)** -3 – (+7)
(i) +12 + (-3)	**(j)** +10 + (-4)	**(k)** +7 + (-9)	**(l)** +4 + (-8)
(m) -7 – (-6)	**(n)** -3 – (-9)	**(o)** +2 – (-10)	**(p)** +1 – (-1)
(q) -7 + (-3)	**(r)** -9 + (-3)	**(s)** -4 – (-9)	**(t)** -2 – (-6)

2 State true (T) or false (F) for the following.

(a) 25 + (+5) = 25 – 5 **(b)** 30 + (-10) = 30 – 10

(c) 15 – (+20) = 15 – 20 **(d)** 40 – (-50) = 40 – 50

3 Calculate:

(a) 6 – 18	**(b)** 9 – 20	**(c)** -5 + 9	**(d)** -3 + 6
(e) -12 + 7	**(f)** -10 + 2	**(g)** -9 – 4	**(h)** -7 – 5
(i) -60 + 30	**(j)** -52 + 21	**(k)** -27 + 84	**(l)** -28 + 93

4 **(a)** -9 – (-6) – (+4) simplifies to:

A -9 – 6 – 4 **B** -9 + 6 + 4 **C** -9 + 6 – 4 **D** 9 + 6 + 4

(b) 14 – (+4) + (-3) is the same as:

A -14 – 4 – 3 **B** 14 – 4 – 3 **C** 14 – 4 + 3 **D** 14 + 4 + 3

5 Calculate:

(a) -6 + 7 + 5	**(b)** -4 + 6 + 5	**(c)** -12 + 3 + 2	**(d)** -11 + 3 + 4
(e) -8 + 5 – 3	**(f)** -6 + 2 – 4	**(g)** -4 + 12 – 4	**(h)** -3 + 10 – 6
(i) -5 – 1 – 6	**(j)** -2 – 5 – 4	**(k)** 36 – 40 – 5	**(l)** 43 – 50 – 2

Understanding

6 Your monthly bank statement shows a balance of $260 at the start of a particular month, followed by the transactions for the month listed below.

Credit	$30
Debit	-$80
Credit	$200
Debit	-$60
Debit	-$500

What is your account balance at the end of the month?

7 A maintenance worker in an office block starts work on the ground floor, and then travels as follows.

Up 10 floors
Down 7 floors
Down 5 floors
Up 4 floors
Down 6 floors
Up 12 floors

(a) What was the highest floor the worker reached?

(b) What was the lowest floor the worker travelled to?

(c) Which floor did the worker finish on?

8 Diana makes and sells soft toys. She keeps records of her profits and losses over a 10-week period. These records are shown in the table. Find Diana's overall profit or loss during this time.

Week	Result	
1	Profit	$120
2	Profit	$25
3	Loss	-$70
4	Profit	$210
5	Loss	-$150
6	Loss	-$180
7	Loss	-$90
8	Profit	$40
9	Loss	-$160
10	Loss	-$95

Reasoning

9 Complete the following addition and subtraction grids. Begin with the number at the top of each column, then either add or subtract the numbers at the left of each row.

(a)

+	-6	+20		31
+4				
		+13		
	+2			
			-23	22

(b)

–	8	-11		
	5			
-7			26	
+5				-30
		-1		

10 A 'magic square' is a square grid of numbers where the sum of each of the rows, columns and diagonals is the same number.

(a) What is the magic sum for this 3 × 3 magic square?

(b) Complete this magic square.

	6	-1
	2	
5		

11 You can make new magic squares by adding or subtracting the same number from every number in an existing magic square.

(a) Make a new 3 × 3 magic square by subtracting 4 from each number in the magic square in the previous question.

(b) What is the new magic sum? Explain why the new magic sum is not 4 less than the old magic sum.

Open-ended

12 In the sport of golf, each hole has a particular number of shots that are expected to get the ball from the tee to the hole. This expected number of shots is called the 'par' for the hole. If you take this number of shots to get the ball in the hole, your score for the hole is 0. If your number of shots for the hole is *less than par*, this is shown on the bottom row of the scorecard by a *negative number*. If your number of shots is *greater than par*, this is shown with a *positive number*.

(a) Carla played nine holes of golf. She scored under par on four holes, par on three holes and over par for the rest. If Carla took a total of 35 shots to complete her round, fill in what her scorecard may have looked like.

Hole number	1	2	3	4	5	6	7	8	9	Total
Par for the hole	4	4	5	3	4	5	4	3	4	36
Shots taken										35
Score compared to par										

(b) Carla played another round of nine holes. Her overall score was -1 (1 under par). What could her scorecard have looked like, if she did not score par (0) on any hole?

13 Find two numbers, one positive, one negative, that have a difference of 31. Find two more such pairs.

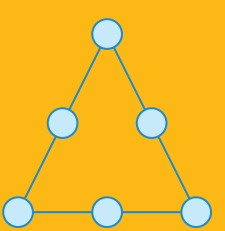
Puzzle

Pattern sums

Copy and complete each of the following patterns. Arrange the numbers given, one in each circle in the pattern, so that the sum of each connected straight line is equal to the given sum value.

1 -4, -3, -2, -1, 0, 1, 2; sum = -3

2 -2, -1, 0, 1, 2, 3; sum = 0



Challenge 2

1 What are the next three numbers in each pattern?

(a) 17, 13, 9, 5, …

(b) -31, -25, -19, -13, …

2 The product of three brothers' ages is 72 and their sum is 14. The youngest of the brothers are twins. What are the ages of the brothers?

3 Find three consecutive numbers (numbers that come one after another, such as 5, 6, 7) so that the sum of the first and third numbers is 172.

4 In a school there are 150 students in Year 7. For a performance in the school hall, chairs are arranged in rows, with the same number of chairs in each row, so that all 150 students are seated with no spare seats. If 10 more chairs were added to each row, everyone could be seated in 4 fewer rows, allowing the people in the back row to be closer to the stage. How many chairs were in each row in the original seating arrangement?

5 What is the sum of all the digits in the numbers from 1 to 100?

6 Jamie needs to know the total amount of money her friends have raised for the 'Save the Koala' fund. All they will tell her is this:

Siena and Levi have collected $130 between them.
If Levi and Marwa pooled their money, they would have $150.
Marwa and Allison have $100 altogether.
Allison and Kaya have just $70 in total.
If Kaya and Siena combined their money, they would have $90.
Tell Jamie what she needs to know.

7 A number less than 100 gives a remainder of 2 when divided by 4, a remainder of 3 when divided by 5, and a remainder of 4 when divided by 6. Find the number.

8 41 is a prime number. If the order of its digits is reversed, it becomes 14, which is *not* a prime number. How many two-digit prime numbers *do* give a prime number when their digits are reversed?

A 8 **B** 9 **C** 10 **D** 11

9 **(a)** How many zeros are at the end of the number given by the following multiplication?

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 19$

A 1 **B** 2 **C** 3 **D** 4

(b) How many zeros are at the end of the number given by the following multiplication?

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 29$

A 3 **B** 4 **C** 5 **D** 6

(c) How many zeros are at the end of the number given by the following multiplication?

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 59$

A 9 **B** 10 **C** 11 **D** 12

(d) How many zeros are at the end of the number given by the following multiplication?

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 99$

A 18 **B** 19 **C** 20 **D** 21

Chapter review 2

Maths literacy

common factor	factor	negative
common multiple	highest common factor (HCF)	positive
composite number	integers	prime factor
co-prime	loss	prime number
deposit	lowest common multiple (LCM)	profit
divisible	multiple	withdrawal

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 A _____ of 6 is 18. The _____ of 6 and 4 is 12.

2 The _____ are all of the positive and negative whole numbers, and zero, which is neither positive nor negative.

3 1, 2, 3, 6, 9 and 18 are the _____s of 18. The _____ of 18 and 27 is 9.

4 A number that is not _____ by any numbers other than 1 and itself is called a _____.

5 A number with more than two factors is called a _____.

6 When you put money into a bank account, you are making a _____.

7 The addition of two negative numbers will always give a _____ answer.

8 If you sell something for less money than you bought it for, you have made a _____.

9 Every whole number greater than 1 can be written as the unique product of its _____s.

10 You make a _____ when you take money out of your bank account.

11 If you sell something for more than you bought it for, you have made a _____.

12 Two numbers are _____ if their highest common factor is 1.

Fluency

1 Find the LCM of: **2.1**

(a) 9 and 6 **(b)** 9 and 12 **(c)** 10 and 15

2 List all the factors of: **2.1**

(a) 36 **(b)** 48 **(c)** 51 **(d)** 100

3 Find the HCF of: **2.1**

(a) 24 and 56 **(b)** 18 and 72 **(c)** 45 and 80

4 Copy the following table and do the divisibility tests. Circle the numbers 2 3 4 5 6 9 10 if the number at the left is divisible by it. **2.1**

5301	2	3	4	5	6	9	10
10 000	2	3	4	5	6	9	10
333 333	2	3	4	5	6	9	10
31 700	2	3	4	5	6	9	10
43 521 820	2	3	4	5	6	9	10

5 State whether each of the following is a prime number or a composite number, and explain why. **2.2**

(a) 5 **(b)** 16 **(c)** 77 **(d)** 276 350

6 By drawing a factor tree or using the 'repeated division' method, express each number as a product of its prime factors. **2.3**

(a) 24 **(b)** 30 **(c)** 88 **(d)** 200

7 Use prime factors to find the HCF of the following. **2.3**

(a) 27 and 36 **(b)** 72 and 96 **(c)** 108 and 240

8 Write an integer to represent the following. **2.4**

(a) 14 degrees below zero **(b)** an altitude of 200 metres

9 State the opposite of: **2.4**

(a) north 5 km **(b)** adding 27

10 Write < or > between the following pairs of numbers to make a true statement. **2.4**

(a) -52 _____ 25 **(b)** 19 _____ -20

11 Write the following numbers in ascending order. **2.4**

(a) -7, 12, 0, -9, 7 **(b)** 4, -4000, 40, 400

12 Calculate: **2.5**

(a) +16 + 2 **(b)** -3 + 18 **(c)** -15 + 5 **(d)** +9 – 3

(e) +1 – 5 **(f)** +16 – 8 **(g)** +7 – 12 **(h)** -14 – 18

13 Calculate: **2.6**

(a) +7 + (-10) **(b)** +9 + (-6) **(c)** -11 – (-4) **(d)** -4 – (-4)

(e) -12 – (-5) **(f)** +5 + (-3) **(g)** -8 – (-5) **(h)** -5 + (-7)

14 Rewrite the following with a single sign between the integers, then evaluate. **2.7**

(a) 9 – (+11) **(b)** -3 + (+10) **(c)** -10 – (-21) **(d)** 8 + (-12)

(e) -4 – (-41) **(f)** -14 + (+28) **(g)** -5 – (+8) – (-2) **(h)** 4 + (-9) + (+2)

Understanding

15 Use the words 'multiple', 'factor' and 'divisible' to complete the following sentences. **2.1**

(a) 45 is _____ by 9, so that makes it a _____ of 9.

(b) 8 is a _____ of 56, so 56 is a _____ of 8.

(c) 27 is _____ by 3, so that makes it a _____ of 3.

16 If 96 lollies are to be divided into packets so that each packet contains the same number, how many lollies can be in each packet? Give all possible combinations. **2.1**

17 Use the symmetry of the number line to help you calculate the following. **2.6**

(a) -31 + 19 **(b)** -54 – 27 **(c)** -22 + (-38) **(d)** -9 – (61)

18 Describe the number line journey you could follow to find the value of: **2.6**

(a) +3 + (-8) **(b)** -6 – (+5)

19 For each pair of numbers, state whether or not they are co-prime. If not, explain why. **2.2**

(a) 11 and 27 **(b)** 51 and 63 **(c)** 14 and 35 **(d)** 24 and 55

20 In the game of indoor cricket, 5 runs are subtracted from a team's score every time a wicket is taken. Here is what happened in the first over of a game: **2.6**

Ball 1: 2 runs
Ball 2: wicket
Ball 3: 1 run
Ball 4: wicket
Ball 5: 1 run
Ball 6: 4 runs

What was the score at the end of the over?

21 There are 84 junior and 108 senior club members at an official club dinner. The dinner organiser wants to have an equal number of junior and senior members at each table. **2.3**

(a) Use prime factors to find the HCF of 84 and 108, and so find the number of tables required.

(b) Use your answer from **(a)** to find the number of junior and senior members at each table.

22 Find the first common multiple of 2, 7 and 9 that is greater than 500. **2.1**

23 **(a)** What is the first prime number after 70? **2.2**

(b) What is the first composite number after 70?

24 Michelle made deposits of $210, $25, $45 and $66 into her bank account during one month, and withdrawals of $35, $56, $214 and $102 during the same period. **2.6**

(a) At the end of the period, had her balance increased or decreased?

(b) By how much had it increased or decreased?

25 Joanna and Petra are on two different ferris wheels, both rotating clockwise. The first wheel takes 25 seconds to make a rotation and the other takes 30 seconds. If Joanna and Petra were both at eye level at the bottom of each of their ferris wheels when they start turning, how many seconds will pass until they are again both at the bottom at eye level? **2.6**

26 Miners in a copper mine are working 900 m underground. They get in a lift and travel a further 250 m down. What depth are they working at now? Write your working and answer using negative integers. **2.6**

Reasoning

27 Copy the following and write <, = or > to make true statements. **2.7**

(a) $4 - (-7) \_\_\_\_ 4 + 7$ **(b)** $-3 + 2 \_\_\_\_ 3 - 2$

(c) $5 + (-3) \_\_\_\_ 5 - (+3)$ **(d)** $-8 - 9 \_\_\_\_ -8 + 9$

28 If you know that a number is divisible by 8, what other numbers do you also know it is divisible by? **2.1**

29 A number between 900 and 1000 has four prime factors: 2, 5, 7 and one other factor. What is the number, and what is the missing factor? **2.1**

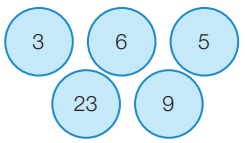
Numeracy practice 2

Non-calculator

1 Alicia is standing at -4 on a large number line placed on the floor. She walks 11 steps in the positive direction. At which number is she standing now?

A -15 **B** -7 **C** 7 **D** 15

2



The sum of the composite numbers in the group shown above is:

A 8 **B** 15 **C** 18 **D** 31

3 A maintenance worker in a city office building gets in a lift in the 3rd basement level (3 floors below ground level) and goes up 11 levels. What floor does the worker get out on?

4 The number 42 written as a product of prime factors is:

A 21×2 **B** $2 \times 3 \times 7$ **C** 1×42 **D** 6×7

5 On a sunny winter's day in Moscow, the temperature at midday was 3°C. By midnight it had dropped to -9°C. The integer that represents this change is:

A -12 **B** -3 **C** 3 **D** 12

Calculator allowed

6 In a remote town, once every week a train passes and every 10 days an aircraft flies overhead. If the train and the aircraft are observed at the town on a certain day, then how many more days will it be until both appear again at the same time?

A 10 **B** 11 **C** 50 **D** 70

7 Vin has $260 in his bank account. During one month, he makes the following transactions.

Deposit: $55
Withdrawal: -$75
Withdrawal: -$33
Deposit: $85
Withdrawal: -$27

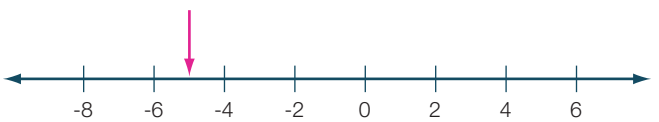
How much does Vin have in his account at the end of the month?

8 The ages (in years) of three people are 65, 39 and 52. The highest common factor of the three ages is:

A 1 **B** 13 **C** 39 **D** 165

9 The arrow is pointing to an integer on the number line.

What number is at this position?



Mixed review A

Fluency

1 Write these integers in ascending order. **2.4**

8, 17, -10, 0, -25, 32, -48

2 Write the following in index form. **1.2**

(a) 9 squared **(b)** $7 \times 7 \times 7 \times 7$ **(c)** 4 cubed

3 List all numbers divisible by both 8 and 6 that are less than 100. **2.1**

4 Write an integer to represent the following. **2.4**

(a) a bank withdrawal of $570 **(b)** a win by 5 points

5 Write < or > between the following pairs of numbers to make a true statement. **2.4**

(a) -27 ____ 14 **(b)** 0 ____ -35

6 Calculate: **1.3**

(a) $8000 \div 200$ **(b)** $1200 \div 4$ **(c)** $45\,000 \div 90$

(d) 30×120 **(e)** 400×1500 **(f)** 2000×5000

7 Use a mental strategy to calculate the following. **1.1**

(a) $4 \times 17 \times 5$ **(b)** $183 + 220$ **(c)** 42×19

(d) 36×11 **(e)** $169 + 71$ **(f)** $5 \times 24 \times 8$

8 Evaluate: **1.5**

(a) $6 \times 4 \div 2 \times 6$ **(b)** $5 + 6 \times 7$ **(c)** $18 + 12 - 7 + 6$

(d) $2 + 5 \times 9$ **(e)** $18 \div 6 - 3$ **(f)** $8 \times (15 - 5)$

9 Calculate the following. **2.5, 2.6**

(a) $-9 + 7$ **(b)** $5 + (-8)$ **(c)** $-3 - (-7)$ **(d)** $-6 - 11$

10 Estimate the answers to the following by rounding to the first digit. **1.4**

(a) 17×93 **(b)** 46×281 **(c)** 337×240

(d) $953 \div 11$ **(e)** $8195 \div 237$ **(f)** $12\,495 \div 5400$

11 Arrange the following numbers in ascending order (from smallest to largest). **2.4**

(a) 5, 0, -15, 10, -5 **(b)** -300, 3, 0, -30, 3000

12 Find the lowest common multiple of: **2.1**

(a) 8 and 12 **(b)** 12 and 16

13 Find the highest common factor of: **2.1**

(a) 36 and 27 **(b)** 64 and 72

14 List the factors of each of these numbers and state whether each number is prime or composite. **2.2**

(a) 18 **(b)** 23 **(c)** 44 **(d)** 79

15 Write each number as the product of its prime factors in index form. **2.3**

(a) 63 **(b)** 48 **(c)** 72 **(d)** 120

16 Simplify the following by writing a single sign between the values, then calculate the answer. **2.7**

(a) -6 + (-7) **(b)** 4 – (-11) **(c)** -2 + (+7)

(d) 22 + (-9) **(e)** -5 – (-10) **(f)** 18 + (+3)

Understanding

17 A submarine 110 m below the surface of the water rises 80 m, then dives 150 m. What depth is it at now? **2.5**

18 Which of the following numbers are: **2.1, 2.2**

(a) prime

(b) perfect squares

(c) powers of 2?

3, 7, 9, 24, 11, 16, 19, 43, 32, 28, 13, 8, 2, 25

19 What is the first perfect cube that is divisible by both 3 and 4? **1.2, 2.1**

20 Calculate an approximate answer for the following by rounding to the first digit, then state whether the actual answer will be higher or lower. **1.4**

(a) 256 × 37 **(b)** 1379 × 24 **(c)** 5498 ÷ 46

21 The Royal Easter Show runs for 7 days. The total attendance at one year's show was 62 982. **1.4**

(a) Approximately how many people per day was this? Use rounding to a convenient multiple of 1000 to calculate your answer.

(b) If each person paid an average ticket price of $12, use your answer from **(a)** to calculate how much money the show organisers made from ticket sales. Use some mental or written strategies to calculate your answer.

Reasoning

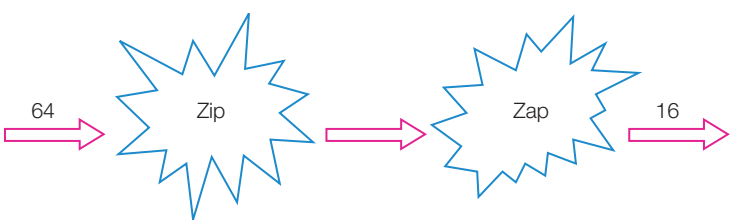
22 On Monday, Kiran withdrew $100 from his bank account at an ATM. On Tuesday, he used his account to pay his $85 phone bill online. On Wednesday, he deposited $250. On Friday, he withdrew another $60 from the account. **2.5**

(a) By the end of the week, did Kiran have more or less money in his bank account than at the start?

(b) How much more or less?

23 The number 64 passes through two 'magic clouds' Zip and Zap. It then emerges as the number 16. **1.2**

Which of the following can describe what Zip and Zap did to the number passing through?



A Zip: square root, Zap: square **B** Zip: cube root, Zap: square

C Zip: square root, Zap: nothing **D** Zip: nothing, Zap: cube root